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# CLASSICAL LOGICS IN COMPUTER SCIENCE

reasoning

inference rule

deduction

algorithm

tautology

proof

Presa Universitară Clujeană

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**CLASSICAL LOGICS**  
**IN COMPUTER SCIENCE**

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## PREFACE

The purpose of this book is to present fundamental concepts and results of classical logics in a formal style and in an explicitly computational way. Applications to automated theorem proving in propositional logic and first-order logic are discussed. The studied proof methods are: the resolution method, the semantic tableaux method and the sequent calculus.

The paper combines the theoretical presentation of classical logics with numerous examples explained and a rich base of proposed exercises.

**Chapter 1** is dedicated to *propositional logic*. The semantic issues discussed are: truth tables, validity, consistency, inconsistency, logical equivalence, logical consequence, normal forms. From a syntactic perspective, propositional logic is introduced as an axiomatic (deductive) system, with the purpose of reasoning modeling.

*First-order (predicate) logic* is the topic of **chapter 2** of the paper. A Hilbert axiomatic system is used to present predicate calculus in a syntactic approach. The semantics of predicate logic is introduced in order to provide a meaning in terms of the modeled universe for each formula from the language. Normal forms, substitutions and unifiers used in predicative resolution are also discussed.

**Chapter 3** treats the *semantic tableaux method*, a refutation proof method. The classic approach through graphical representation using a binary tree is very suggestive. The formulas are decomposed in order to determine their models.

**Chapter 4** presents *the sequent calculus*, an improvement of the natural deduction system. As a syntactic and direct proof method, it is used to check the validity/derivability and non-validity/non-derivability in propositional logic and first-order logic.

The topic of **chapter 5** is *resolution*, a syntactic and refutation proof method, very efficient and easily to implement. Resolution is introduced as an axiomatic (formal) system and as a procedure. In order to increase the efficiency of the resolution process, the strategies (deletion, set-of-support, unit preference, level saturation, linear) and the refinements (lock, linear) of resolution are studied.

By its content, this book is useful to all those interested in classical logics, fundamental in computer science. Professionals in computer science are offered a theoretical basis in the applicative direction of building automated proof systems used in mathematics, software engineering, intelligent agents, robotics, natural language, artificial vision.

We wish to acknowledge our deep appreciation to Prof. Dr. Doina Tătar for many valuable scientific discussions and guidance during all the years of study and research in the field of classical logics. Special thanks for all her constructive comments made during the preparation of this paper.

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Mihaiela-Ana Lupea  
Andreea-Diana Mihiş

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modus ponens

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modus tollens



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